Hidden Symmetries in One-Dimensional Photonic Crystals

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One-dimensional photonic crystals (PCs) are systems with N layers of dielectric media varying in only one direction. They allow us to control the propagation of light, which has useful applications in creating optoelectronic devices. In this study, we investigate the optical properties of PCs in which each layer is one of two dielectric media, with thickness one-quarter the wavelength of light corresponding to a central frequency in that medium. Tavakoli and Jalili[1] found that certain self-similar PCs can produce desirable optical properties, such as a very sharp peak in the transmission spectrum or a high electric field enhancement. However, these are only very specific sequences out of all possible 2^N. Using the transfer matrix method, the transmission probability T is calculated for all possible 2^N sequences for small N. Unexpectedly, it was found that instead of 2^N different values of T at the central frequency, there is only a small number of discrete values. For example, with $N = 8$, there are $2^8 = 256$ possible sequences, but only 5 distinct T values. In physics, high degeneracy generally implies the existence of hidden symmetry operations. We found such symmetries and proved that they do not change T. Additionally, analytical formulae were derived for the T values and degeneracy at each as functions of N and a sequence's total "charge", which we shall define. By understanding the origin of these hidden symmetries and patterns, we can create more efficient algorithms for finding and designing optimal sequences of PCs for large N.

[1] Tavakoli, M., and Y. S. Jalili. "One-dimensional Fibonacci fractal photonic crystals and their optical characteristics." *J. Theor. Appl. Phys.* 8.113 (2014).

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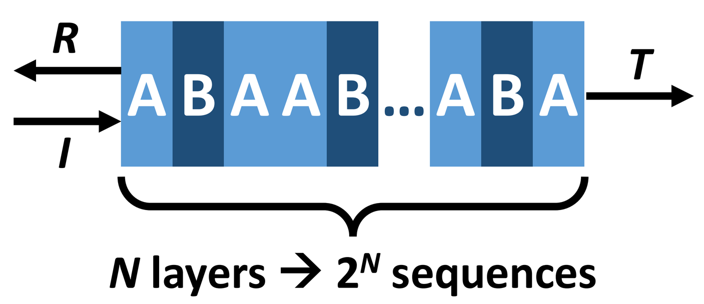
Introduction

1-D Photonic Crystal (PC)



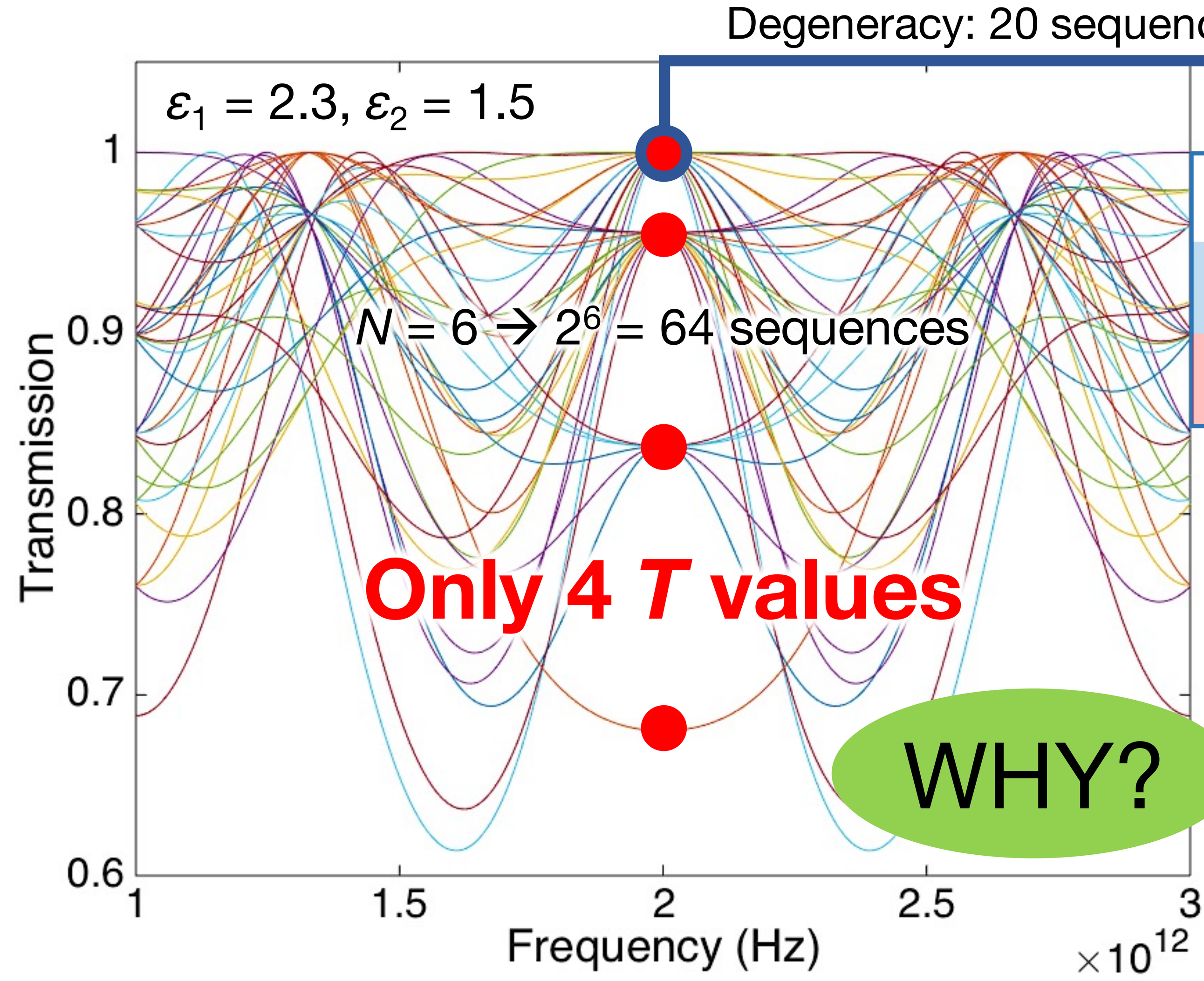
Transfer matrix method

Matching: $[AB]$
Propagation: $[A]$
Total: $M_{seq} = [VA][A][AB][B][BA] \dots$
Reynolds et al. (2016)

- A, B: two dielectric media (ε_1, ε_2)
- Design PC for THz optical filter
- Goal: sharp peak with high T
- How? Transfer matrix method
- Consider all possible 2^N sequences

Unexpected discovery!

Degeneracy: 20 sequences with same T



of T values
Even: $N/2 + 1$
Odd: $N + 1$

Central frequency f_0 = 2 THz

Layer thickness $d_i = \frac{\lambda_0}{4n_i} = \frac{c}{4f_0\sqrt{\varepsilon_i}}$ $d_1 = 2.47 \times 10^{-5}$ m, $d_2 = 3.06 \times 10^{-5}$ m

Purpose

To understand how the PC sequence affects the transmission probability T

Results and Discussion

PROVEN

What do they have in common?

AAAAAA
AAAABB
AABBAA
AABAAB
ABAABA
⋮
BBBAAB
BAAAAB
BBBBBB

Symmetry

Mirror Even | Odd

ABABBA | ABBABA

Inversion Even

ABABBA → BABAAB

Cyclic Even

ABABBA → BBAABA

+ double perm, pair inversion

Degeneracy

How many at each T?

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Even:

$\binom{N}{\frac{N}{2}}$ for $|q| = 0$

$2\binom{N}{\frac{N}{2}+|q|}$ for $|q| = 1, 2, \dots, \frac{N}{2}$

Odd:

$\binom{N}{\frac{N+1}{2}+q}$ for $q = 0, \pm 1, \dots, \pm\frac{N-1}{2}, -\frac{N+1}{2}$

Formula for T

Definition of "Charge" (q)

Physical meaning of "charge"?

Even

ABAAABBA

AB | AA | AB | BA
+1 | 0 | +1 | -1

→ q = +1

$$T = \frac{4(\varepsilon_1\varepsilon_2)^{|q|}}{\left(\varepsilon_1^{|q|} + \varepsilon_2^{|q|}\right)^2}$$

Odd

BABBBAX

BA | BB | BA | A / B
-1 | 0 | -1 |

q =
A: 0 → -2
B: -1 → -3

$$T = \frac{4\varepsilon_1^{|q+1|}\varepsilon_2^{|q|}}{\left(\varepsilon_1^{|q+1|} + \varepsilon_2^{|q|}\right)^2}$$

Conclusion

- T discrete at f_0, depends only on "charge"
- Symmetry operations don't change T
- Degeneracy explained by combinatorics

Future Work

- Analyze symmetries using group theory
- Investigate how shape of T spectrum depends on sequence (e.g. sharp peak)
- Find and design optimal PC sequences
- More on "charge" (secret!)

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