Observing and Modelling Synchronization Phenomena in Oscillatory Systems

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Synchronization, a phenomenon in which two or more objects in a system act in unison, is prevalent throughout nature. Examples include an audience's applause, which synchronizes after a short period to create a single, large, regular rhythm of clapping[1]; the illumination provided by many fireflies[2], which may eventually begin emitting light in unison. Interest in the synchronization phenomenon in physics began with observations of pendulum synchronization on a ship by Dutch physicist Christiaan Huygens in the 17th century[3]. Synchronization is also prevalent in solid state physics, and is an important component of studies in plasmons[4], and the coherent phonon phenomenon[5] in a system of carbon nanotubes with synchronized radial breathing modes. In order to understand synchronization in these types of systems, we consider a model which consists of a group of oscillators interacting with each other through an oscillating substrate. Each oscillator is modelled by a mass m attached to a spring with a spring constant of k and a damping factor of γ. We expect that these parameters affect the synchronization. Using analytical mechanics, we determine the equations of motion for each of the small particles' positions as a function of time through numerical calculations by solving coupled differential equations using the Runge-Kutta approximation method, implemented through Python programming. Defining synchronization time to be the time it takes for a system of small oscillators to have the same displacement from equilibrium, we've found a link between k, m, and γ and the synchronization time.

References

[1] Z. Néda, E. Ravasz, Y. Brechet, T. Vicsek, & A.-L. Barabási. "Self-organizing processes: The sound of many hands clapping." *Nature.* 403, 849-850 (24 February 2000)

[2] Buck, John. "Synchronous Rhythmic Flashing of Fireflies. II." *The Quarterly Review of Biology* 63.3, 265-289 (1988)

[3] M.Bennett, M. F. Schatz, H. Rockwood, & K. Wiesenfeld. "Huygens's clocks." *Proceedings of the Royal Society A.* 458. 563-579 (2002)

[4] D. Pines, D. Bohm, "A collective description of electron interactions: II. Collective vs individual particle aspects of the interactions." *Phys. Rev.* 85.2. 338 (15 January 1952)

[5] G. D. Sanders, A. R. T. Nugraha, K. Sato, J-H Kim, J. Kono, R. Saito, & C. J. Stanton. "Theory of coherent phonons in carbon nanotubes and graphene nanoribbons." *J. Phys: Condens. Matter*. 25 (2013)

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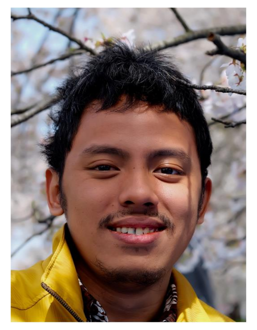

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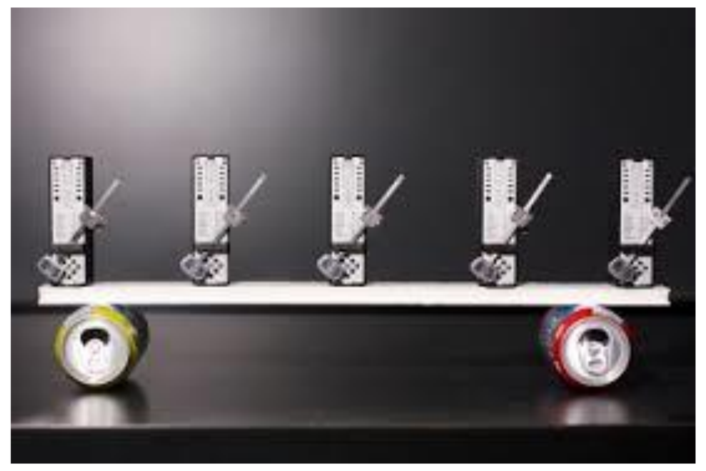
Introduction

Synchronization?

- Objects acting in unison
- Nature
  - Fireflies and Fish
- Solid State Physics
  - Coherent Phonon Carbon Nanotubes
  - Plasmons



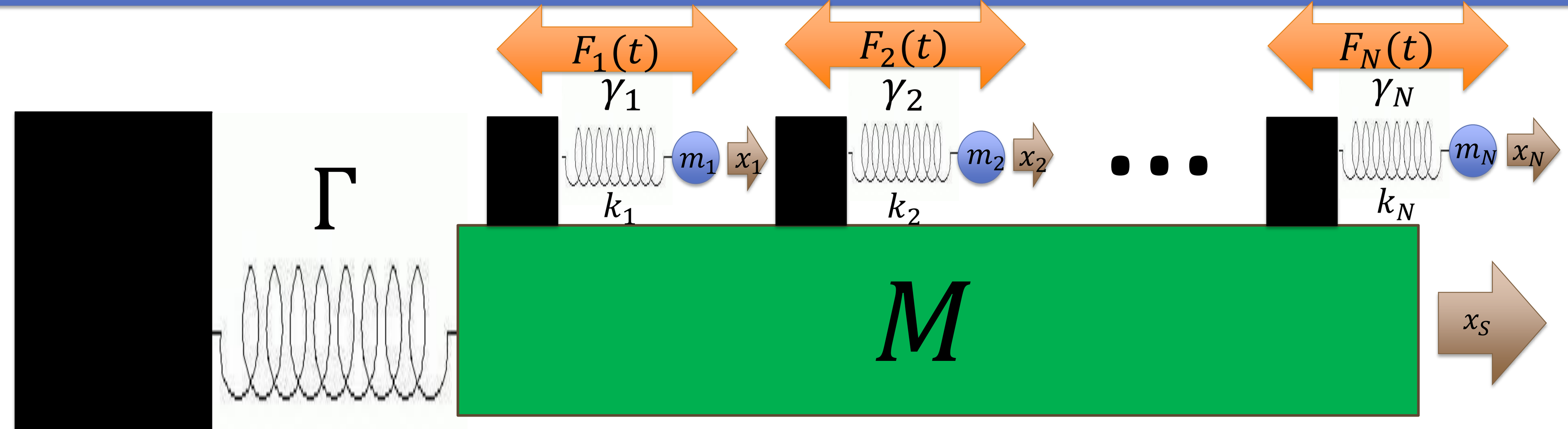
Can we control the time to synchronize?

YES!

Purpose

Observe a model and find what parameters affect time to synchronize.

Modeling System and Methods



K Apply Lagrangian Mechanics $L = T - V$

$$T = \frac{M\dot{x_s}^2 + \sum_{i=1}^{N} m_i(\dot{x_i}+\dot{x_s})^2}{2}$$

$$-V = -\frac{(Kx_s^2 + \sum_{i=1}^{N} k_i(x_i - x_s)^2}{2}$$

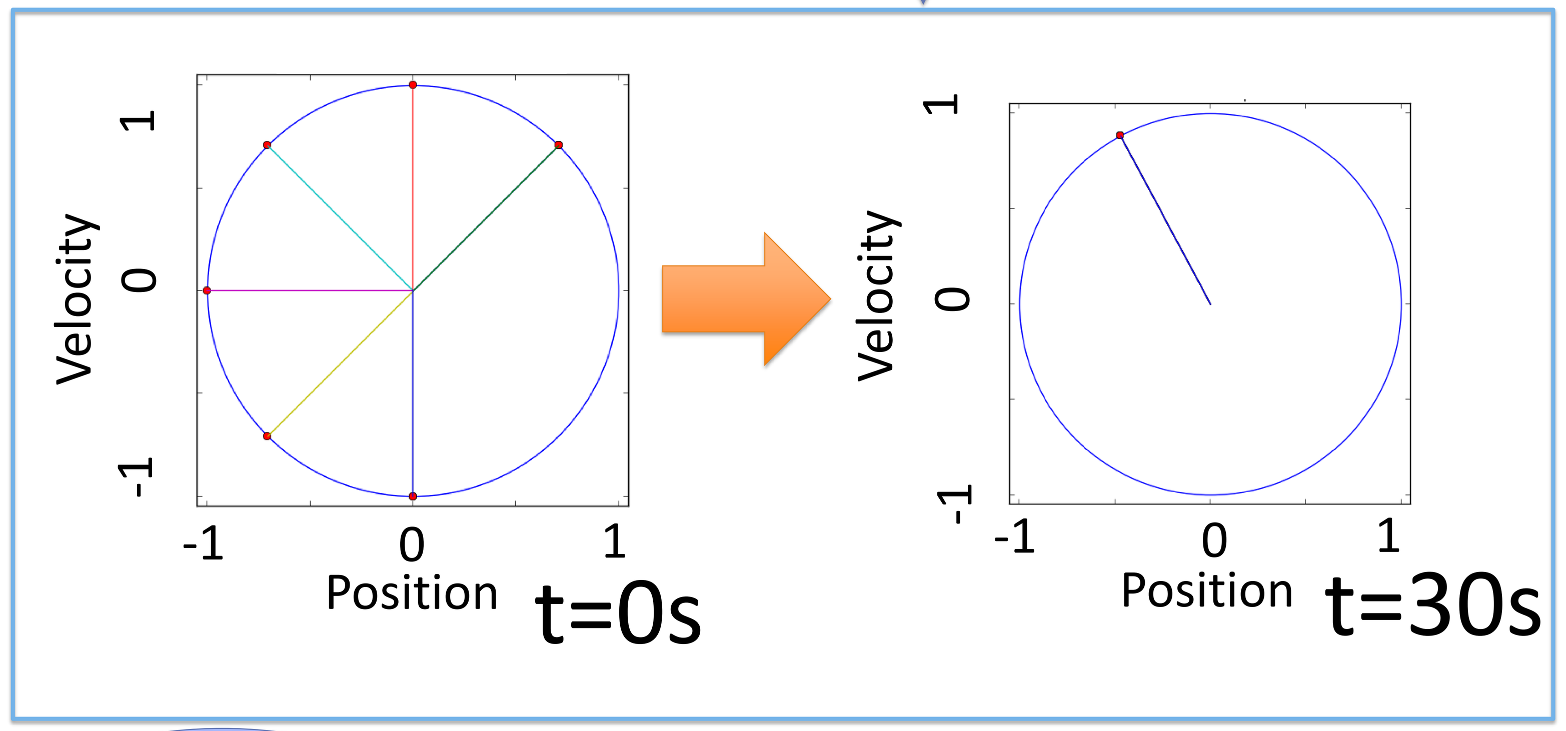
Results and Analysis

Analytically Impossible!

Runge-Kutta method

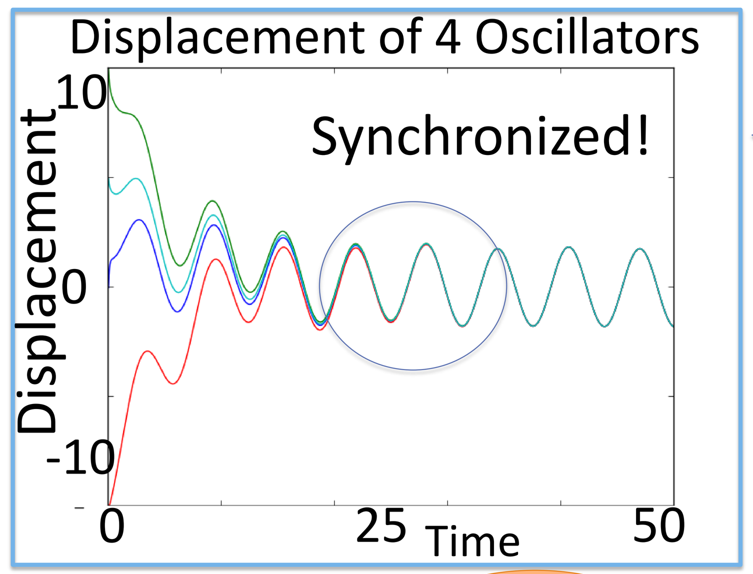
How to define synchronization?

Phase

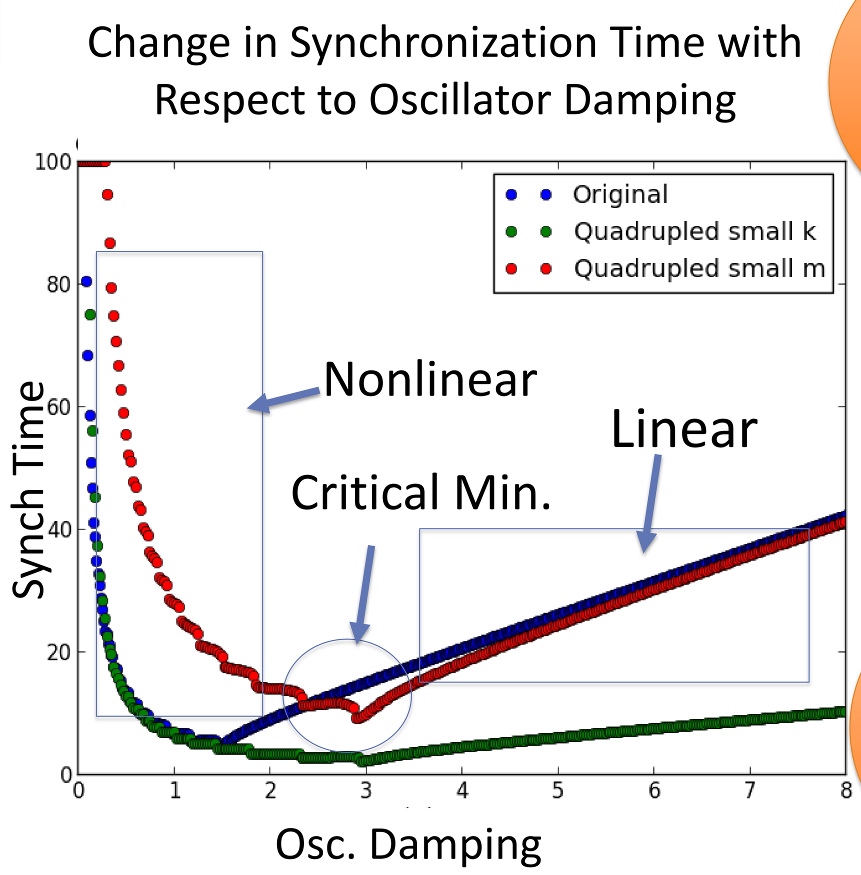


How to measure Synchronization?

$$s_N = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

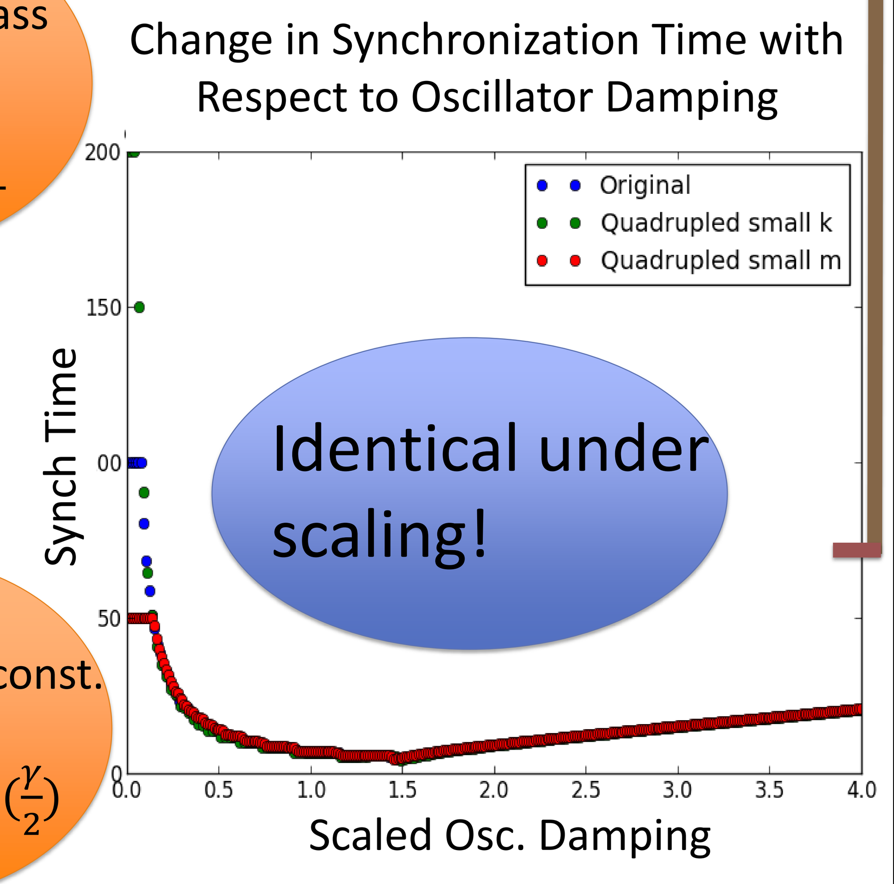


Amplitude



Quadrupled mass scaling:
$\tau(\gamma) \rightarrow \frac{\tau(\frac{\gamma}{2})}{2}$

Quadrupled k const. scaling:
$\tau(\gamma) \rightarrow 2 * \tau(\frac{\gamma}{2})$



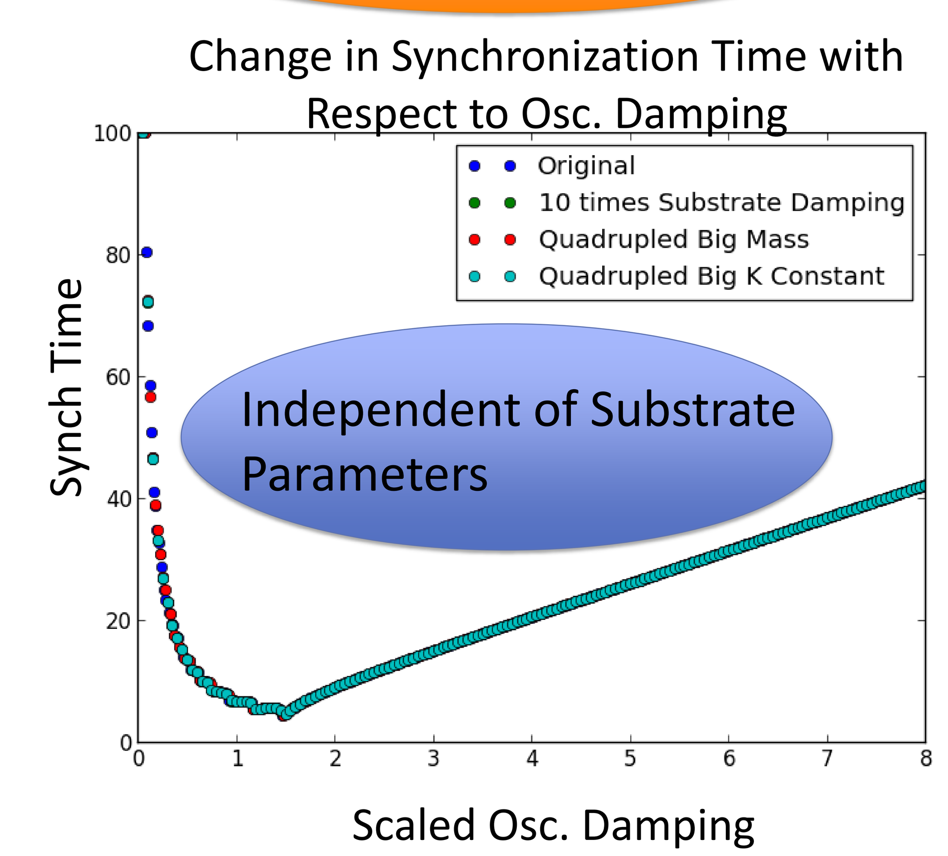
$$\tau(\gamma, ck, dm) = \frac{\sqrt{d}}{\sqrt{c}}\,\tau\left(\frac{\gamma}{\sqrt{cd}}, k, m\right)$$

$\gamma > 2\sqrt{mk}$

$$\tau(\gamma, k, m) = A\frac{\gamma}{k}$$

$\gamma < 2\sqrt{mk}$

$$\tau(\gamma, k, m) = B\frac{m}{\gamma}$$



Conclusions

- Substrate parameter independent
- Fastest synchronization at $\gamma = 2\sqrt{mk}$
- Synchronization becomes either mass or k constant independent.

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